TRANSIENT STABILITY ANALYSIS

1. Introduction

The power system is continually being subjected to changes or disturbances of various forms: faults, load changes, connection/disconnection of generators etc. As the power network is a complex electro-mechanical system, these events cause oscillations in the speed and angles of machines and in power flows along the lines. Transient stability analysis is the study of the system in response to these changes and is used to determine if the system will be stable after a given disturbance. For proper operation of the system, it is essential to ensure that after a given disturbance, the system settles down to a new, stable condition.

Earlier, the response of a load/generation area to changes in load was considered and the frequency deviation from the rated value was predicted. For power/frequency control analysis, it was possible to linearise the system representation about its operating point and hence the application of the Laplace Transform was possible. However, transient stability analysis is generally concerned with the analysis of the effects of major disturbances, such as line faults.

An allied problem is the study of steady-state (or dynamic) stability. This is used to determine if a certain system condition is stable and to look at the response for small fluctuations. As with load/frequency control, linear analysis can be used for steady-state stability analysis. On the other hand, transient stability analysis involves major disturbances. If a three-phase fault occurs close to a machine, then the ability of that machine to transfer power changes from possibly 100% to zero. Hence, we must consider a much wider response range and the analysis is essentially non-linear.

For that reason, the transient analysis of a power network in adequate detail is a formidable task and is one of the major studies involved in power systems analysis. In the past, considerable simplifying assumptions were used to allow for some analysis of transient stability. This often involved considering one machine connected to an infinite system (constant voltage and frequency) through a transmission line. With the improved digital computer resources, it is now possible to investigate the transient stability of extensive systems consisting of many machines.

The significant components for transient stability are:

1. The network before, during and after the transient disturbance.
2. The loads and their characteristics.
3. The parameters of the synchronous generators.
4. The excitation systems of the generators.
5. The turbine and speed governors.

2. Single Machine/Infinite Bus System

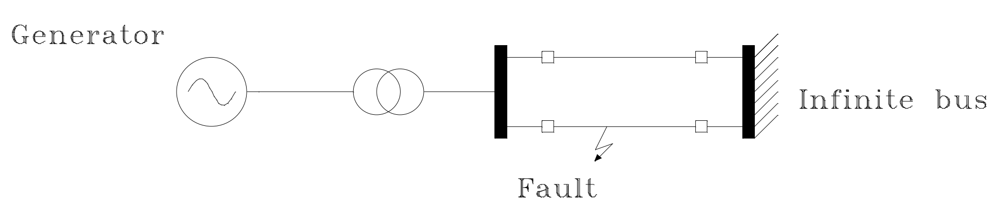

Figure 1 Single machine/Infinite bus system

Figure 1 shows a single machine connected to an infinite bus through parallel transmission lines. The lines have circuit breakers installed at each end. A fault occurs at some point along the second transmission line. The objective is to determine the behaviour of the system before the fault, during the fault and after the fault has been cleared by opening the faulted transmission line. To do this we need to consider the representation of both the electrical and mechanical aspects of this system.

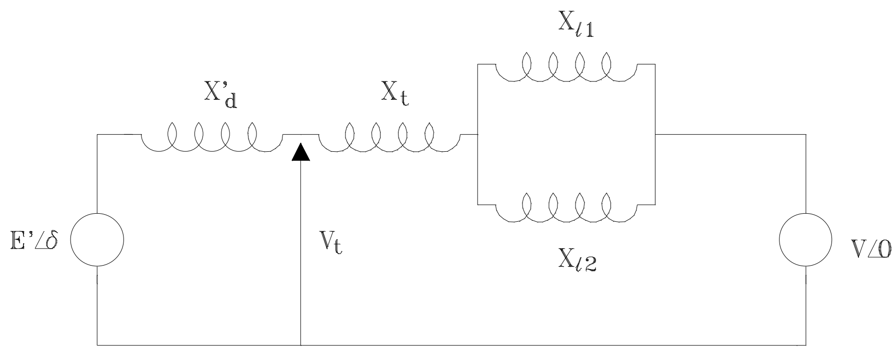

Figure 2 Representation of system

Figure 2 shows the equivalent circuit of the network. The synchronous is represented as a constant voltage E' behind the transient reactance $X_d^{'}$. The voltage behind transient reactance makes an angle of δ with the infinite bus voltage and this is equal to the machine rotor angle. The terminal voltage of the generator is V_t and the transformer reactance is X_t. The reactances of the lines are $X_{\ell 1}$ and $X_{\ell 2}$.

3. The Swing Equation

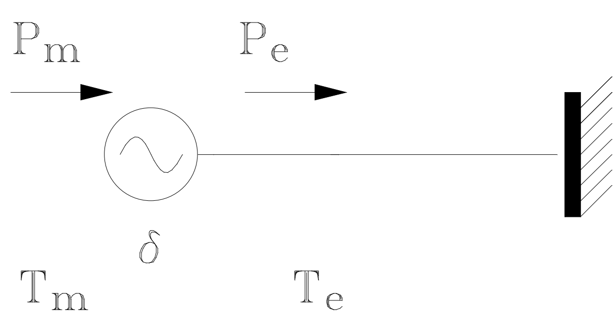

Figure 3 Representation of Generator

The swing equation describes the behaviour of the machine to disturbances in the network. Figure 3 shows a generator of inertia J which input mechanical torque of T_m and output electrical torque T_e. The angular position θ of the machine is given by:

$$\theta = \omega t + \delta$$

where: ω = angular speed or rotation
δ = rotor angle with respect to reference (infinite bus)

Considering the angular position:

$$\frac{d\theta}{dt} = \omega + \frac{d\delta}{dt}$$

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}$$

where the angular speed of rotation is constant.

The acceleration torque on the machine is given by:

$$T_a = T_m - T_e = J\frac{d^2\theta}{dt^2} = J\frac{d^2\delta}{dt^2}$$

or in terms of power:

$$P_a = P_m - P_e = \omega J\frac{d^2\theta}{dt^2} = M\frac{d^2\delta}{dt^2}$$

where M is the angular momentum and it assumed that the change in speed is small.

If damping is included, then:

$$P_m - P_e = M\frac{d^2\delta}{dt^2} + P_d = M\frac{d^2\delta}{dt^2} + K_d\frac{d\delta}{dt}$$

Multiplying by $\omega/2$ gives:

$$\frac{1}{2}\omega(P_m - P_e) = \frac{1}{2}\omega^2 J\frac{d^2\delta}{dt^2}$$

and $\frac{1}{2}\omega^2 J$ represents the kinetic energy of the generator.

If we divide by the base MVA, S_B, we have:

$$\frac{1}{2}\omega\left(\frac{P_m}{S_B} - \frac{P_e}{S_B}\right) = \frac{\frac{1}{2}\omega^2 J}{S_B}\frac{d^2\delta}{dt^2}$$

$$\frac{1}{2}\omega(P_{m,pu} - P_{e,pu}) = H\frac{d^2\delta}{dt^2}$$

H is the inertia constant and is equal to the kinetic energy of the generator divided by its power rating. Hence, the units of H are MJ/MW or seconds and the typical machine has an inertia constant between 2 and 10 seconds. If we now drop the pu (per unit notation is assumed) we have:

$$P_m - P_e = \frac{2H}{\omega}\frac{d^2\delta}{dt^2} = \frac{H}{\pi f}\frac{d^2\delta}{dt^2}$$

or with damping:

$$P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta}{dt^2} + K_d \frac{d\delta}{dt}$$

Transient stability analysis involves the solution of this differential equation - the swing equation. There are two significant aspects to transient stability:

- the input mechanical power P_m
- the output electrical power transferred to the load or network P_e

4. Generator Configurations

4.1. TWO MACHINES CONNECTED IN PARALLEL

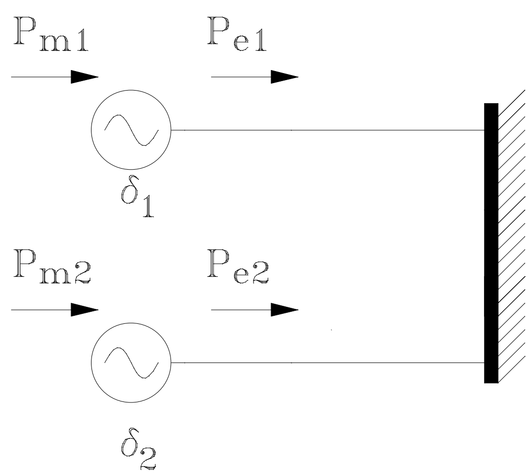

Figure 4 Two machines connected to a common bus

As shown in Figure 4, if we have two machines connected in parallel and supplying the same load, the swing equations are given by:

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$
$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

If the machine swing together, or if

$$\delta_1 = \delta_2 = \delta$$

then

$$\frac{(H_1 + H_2)}{\pi f}\frac{d^2\delta}{dt^2} = (P_{m1} + P_{m2}) - (P_{e1} + P_{e2})$$

This assumption can be used to simplify a system if a number of machines are connected to the same bus.

4.2. TWO MACHINES ARRANGED AS GENERATOR AND LOAD

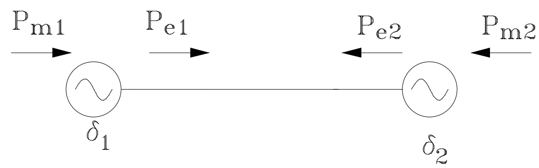

Figure 5 Two machines connected as generator and motor

Consider the two machines shown in Figure 5. One machine is acting as load and the other is acting as generator. The swing equations are:

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

or:

$$\frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2} = \frac{1}{\pi f}\left[\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2}\right]$$

If we take the difference between the machine angles, $\delta_{12} = \delta_1 - \delta_2$, then

$$\frac{P_{m1}H_2 - P_{m2}H_1}{H_1H_2} - \frac{P_{e1}H_2 - P_{e2}H_1}{H_1H_2} = \frac{1}{\pi f}\frac{d^2\delta_{12}}{dt^2}$$

$$\frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2} - \frac{P_{e1}H_2 - P_{e2}H_1}{H_1 + H_2} = \frac{1}{\pi f}\left[\frac{H_1H_2}{H_1 + H_2}\right]\frac{d^2\delta_{12}}{dt^2}$$

$$P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta_{12}}{dt^2}$$

where:

$$H = \frac{H_1H_2}{H_1 + H_2}$$

$$P_m = \frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2}$$

$$P_e = \frac{P_{e1}H_2 - P_{e2}H_1}{H_1 + H_2}$$

If there are no losses in the system, then

$$P_{m1} = -P_{m2}$$

$$P_{e1} = -P_{e2}.$$

If one machine approximates an infinite bus, then $H_2 \rightarrow \infty$ and

$$P_{m1} - P_{e1} = \frac{H_1}{\pi f}\frac{d^2\delta_{12}}{dt^2}$$

Clearly, in assessing the transient stability, the angle difference is most important when considering two or more machines.

5. Transmitted Power

To investigate the swing equation we need an expression which describes the power flow from the machine to the load or the infinite bus. Consider the following simplified circuit:

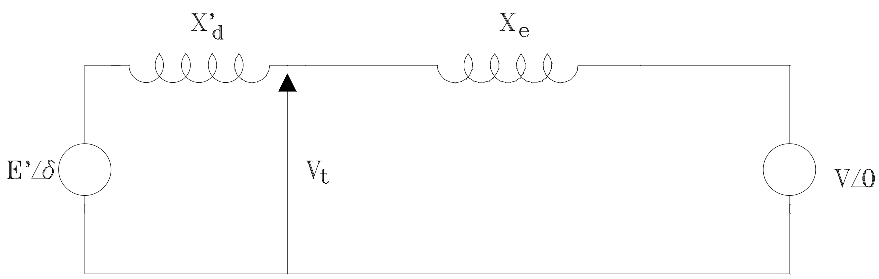

Figure 6 Equivalent circuit representation of network

X_e is the equivalent reactance of the network. E' is the transient voltage behind transient reactance and $X_d^{'}$ is the transient reactance. Figure 7 shows the phasor diagram of the circuit.

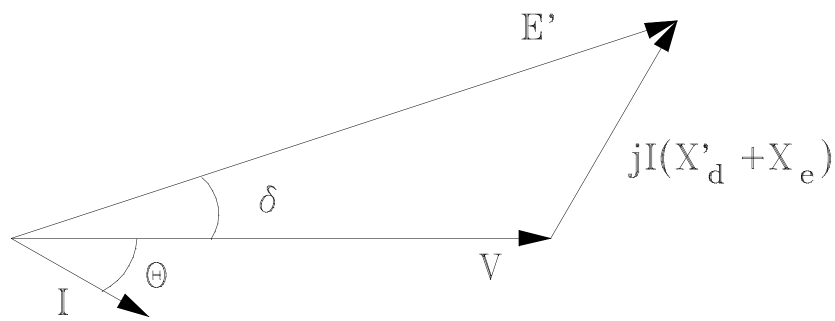

Figure 7 Phasor diagram

The active power being transferred to the infinite bus is given by:

$$P_e = \mathrm{Re}\left[E' I^*\right] = \mathrm{Re}\left[E'\angle\delta\left[\frac{E'\angle\delta - V}{j(X'_d + X_e)}\right]\right]^*$$

$$= \frac{E'V}{X'_d + X_e}\sin\delta$$

Problem:

For a machine connected to an infinite bus, calculate the rotor angle and the equation describing the power transfer to the infinite bus if the machine has a transient reactance of 0.2 pu, the infinite bus voltage is 1 pu and the power being delivered is 0.8 pu at 0.8 pf lag.

If we substitute the expression for the electrical power into the swing equation, we have:

$$P_m = \frac{H}{\pi f}\frac{d^2\delta}{dt^2} + \frac{E'V}{X'_d + X_e}\sin\delta$$

This non-linear differential equation is solved to calculate the transient behaviour of the machine due to a fault. For simplified analysis, the mechanical power input and the voltage behind transient reactance are assumed constant.

6. Linearisation of Swing Equation

For small changes in the rotor angle δ, the swing equation can be linearised. For example, if δ changes to δ+Δδ, then

$$\begin{aligned}\sin(\delta + \Delta\delta) &= \sin\delta\cos\Delta\delta + \cos\delta\sin\Delta\delta \\ &= \sin\delta\cos\delta_\Delta + \cos\delta\sin\delta_\Delta\end{aligned}$$

and if δ_Δ is sufficiently small

$$\cos\delta_\Delta \approx 1, \quad \sin\delta_\Delta \approx \delta_\Delta$$

The power transferred to the infinite bus changes to:

$$P_e \rightarrow P_e + P_{e\Delta} = \frac{E'V}{X'_d + X_e}\left[\sin\delta + \delta_\Delta\cos\delta\right]$$

Comparing the swing equations gives:

$$P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta}{dt^2} + K_d\frac{d\delta}{dt}$$

$$P_m - (P_e + P_{e\Delta}) = \frac{H}{\pi f}\frac{d^2(\delta + \delta_\Delta)}{dt^2} + K_d\frac{d(\delta + \delta_\Delta)}{dt}$$

Subtracting one equation from the other gives:

$$\frac{H}{\pi f}\frac{d^2\delta_\Delta}{dt^2} + K_d\frac{d\delta_\Delta}{dt} + \frac{E'V\cos\delta}{X'_d + X_e}\delta_\Delta = 0$$

This differential equation is now linear. The synchronising power coefficient is defined as:

$$S_p = \frac{E'V\cos\delta}{X'_d + X_e}$$

and the roots of the characteristic equation are given by:

$$s_1, s_2 = \frac{-K_d\pi f}{2H} \pm \sqrt{\left(\frac{K_d\pi f}{2H}\right)^2 - \frac{S_p\pi f}{H}}$$

If damping is assumed to be zero, then

$$s_1, s_2 = \pm\sqrt{-\frac{S_p\pi f}{H}}$$

and if S_p is +ve, the roots are given by:

$$s_1, s_2 = \pm j\sqrt{\frac{S_p \pi f}{H}} = \pm j\omega_n$$

and the system responds with a sustained oscillation at the natural angular frequency ω_n. If S_p is -ve, then one pole is in the right-hand-side of the complex pole-zero plane and the system is unstable. Hence, the synchronising power coefficient must be +ve to ensure steady-state stability. We can see that this means that the system is stable if the operating rotor angle is between -90° and +90°.

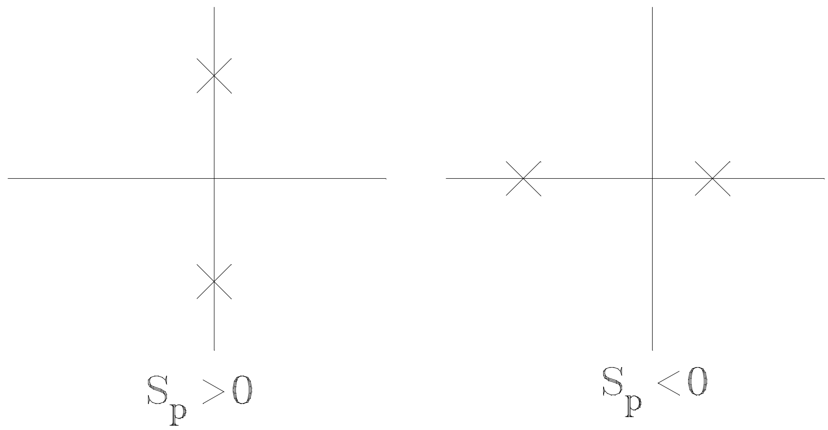

Figure 8 Pole plot for single machine connected to infinite bus

Problem:
A machine with inertia constant of 5 seconds is operating with a voltage behind transient reactance of 1.05 pu and is connected to an infinite bus of 1 pu voltage. The total impedance, including transient reactance is 0.2 pu. The damping constant is 0.5. If the machine is operating at 0.5 pu power output, calculate the natural frequency of oscillation for small disturbances and the damping ratio.

7. Analysis of Transmission Network

To investigate the transient behaviour of the system, we need to solve the transmission network for conditions before the fault, during the fault and after the fault has been cleared. Consider the system shown in Figure 2. For the prefault situation, the power flow from the machine to the infinite bus is given by:

$$P_{e,prefault} = \frac{E'V}{X'_d + X_t + \frac{X_{\ell 1} X_{\ell 2}}{X_{\ell 1} + X_{\ell 2}}} \sin\delta$$

If $X_{\ell 1} = X_{\ell 2} = X_\ell$, then:

$$P_{e,prefault} = \frac{E'V}{X'_d + X_t + X_\ell / 2} \sin\delta$$

When a fault occurs on transmission line 2 at a distance of *a* pu from the infinite bus, then the circuit can be redrawn as shown in Figure 9.

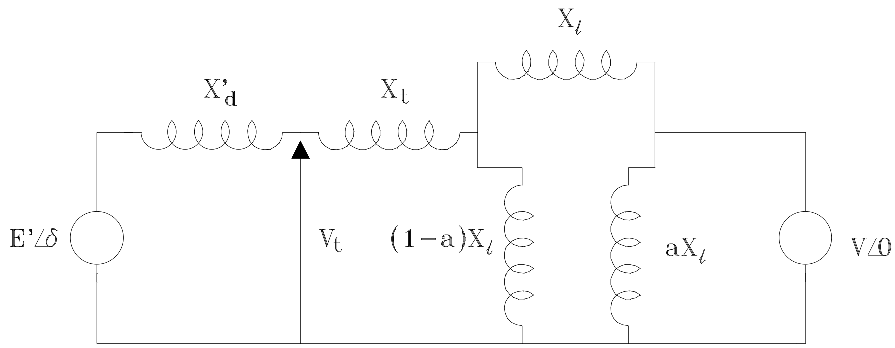

Figure 9 Network with fault

We can simplify this network by applying the star-delta transformation as shown in Figure 10.

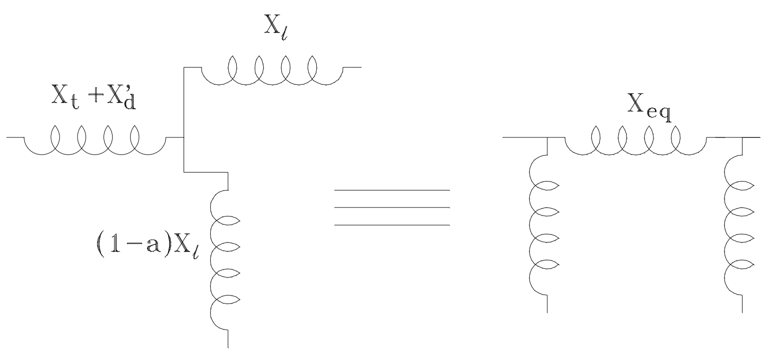

Figure 10 Star-delta transformation

The equivalent reactance in the delta is given by:

$$X_{eq} = \frac{(X'_d + X_t)X_\ell + (1-a)X_\ell^2 + (X'_d + X_t)(1-a)X_\ell}{(1-a)X_\ell}$$

If the fault occurs at the mid-point and a=0.5, then:

$$X_{eq} = 3(X'_d + X_t) + X_\ell$$

and the power flow is given by:

$$P_{e,fault} = \frac{E'V}{3(X'_d + X_t) + X_\ell}\sin\delta$$

Alternatively, the Thevenin equivalent of the infinite bus voltage as seen from the line terminal of the transformer could be used to produce the same result. The fault is cleared by opening both circuit breakers on the faulted line. In this case the power flow is given by:

$$P_{e,post-fault} = \frac{E'V}{X'_d + X_t + X_\ell}\sin\delta$$

Problem:
A generator, connected to an infinite bus via two identical parallel transmission lines has the following data:

H=5 seconds	X'_d *=0.2pu*	X_ℓ *=0.4pu*
V_t *=1.05pu*	X_t *=0.1pu*	P_e *=0.8pu*

A fault occurs at time t=0 at the mid-point of one of the lines and is cleared at time t=t_c*. Develop the differential equations for the behaviour before the fault, during the fault and after the fault has been cleared.*

8. Equal Area Criterion

One method of investigating the transient stability behaviour of a single machine/infinite bus system is to apply the Equal Area Criterion. The method does not solve for the rotor angle, rather it tells us the maximum angle which the machine can advance to before the fault is cleared in order to preserve transient stability. Consider the following swing equation:

$$P_a = P_m - P_e = \frac{H}{\pi f}\frac{d^2\delta}{dt^2}$$

This may be written as:

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}P_a$$

Also:

$$\frac{d}{dt}\left\{\left[\frac{d\delta}{dt}\right]^2\right\} = 2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2}$$

and therefore the swing equation can be written as:

$$\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}P_a\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = \frac{2\pi f}{H}P_a\frac{d\delta}{dt}$$

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f}{H}\int_{\delta_0}^{\delta}P_a d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2\pi f}{H}\int_{\delta_0}^{\delta}P_a d\delta}$$

Therefore, when $\frac{d\delta}{dt} = 0$, the machine rotor angle is no longer increasing, δ has reached a maximum (or minimum) and this occurs at an angle δ_m where:

$$\int_{\delta_0}^{\delta_m}P_a d\delta = 0$$

This means at the integral of the accelerating power (with respect to the rotor angle) must equal zero to ensure stability. When that integral has reached zero, then the rotor angle has reached a maximum (minimum) and will start decreasing (increasing). This criterion can be used to assess the stability of a single machine/infinite bus system. We can also view this by saying that the machine rotor angle will reach a maximum value when the area of the accelerating power equals the area of the decelerating power.

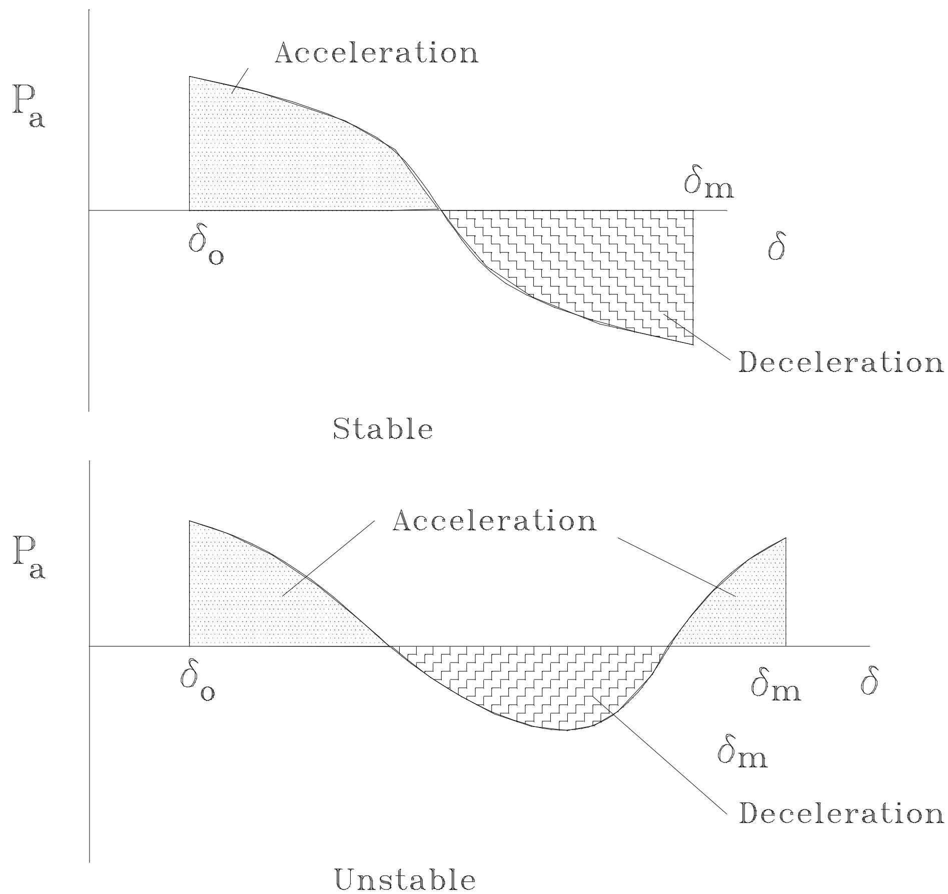

Figure 11 Acceleration and deceleration areas

The application of the equal area criterion is shown in Figure 11. In the first case, the fault occurs at an angle δ_0 and the machine begins to accelerate. The fault is cleared at some subsequent time (and angle) and the machine begins to decelerate and reaches a maximum angle δ_m when the acceleration area and deceleration area are equal. We can see that in this case the machine is stable as the fault was cleared at a time which allowed the machine a sufficient interval to decelerate. In the second case, the acceleration area exceeds the deceleration area and the machine is unstable. This is because the time taken to clear the fault was excessive.

The acceleration area can be determined from the power angle curve. Figure 12 shows the power-angle curves for the system described earlier. The prefault, fault and postfault power curves are shown together with the constant mechanical power. The accelerating power is equal to the difference between the electrical power curve and the mechanical power input.

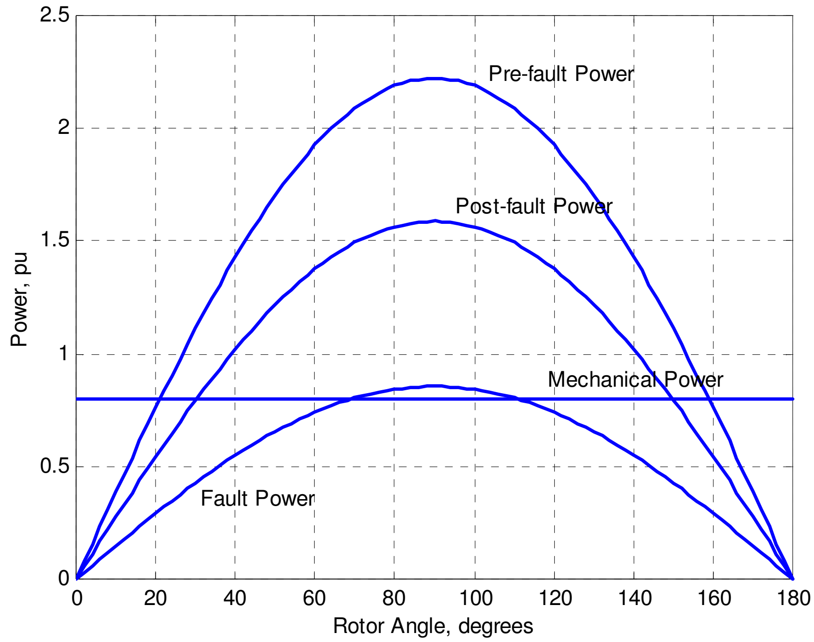

Figure 12 Power-angle curves

The critical clearing angle is the maximum angle before which the fault must be cleared to ensure that the system remains stable. The critical clearing angle can be calculated by applying the equal area criterion. Considering the earlier example and Figure 12, then

$$\int_{\delta_0}^{\delta_c} \left(P_m - P_{fault}\right) d\delta = \int_{\delta_c}^{\delta_m} \left(P_{postfault} - P_m\right) d\delta$$

where δ_0 is the initial rotor angle, δ_c is the critical clearing angle and δ_m is the maximum angle to which the machine can swing to and remain stable. The above equation can be written as:

$$\int_{\delta_0}^{\delta_c} \left(P_m - \frac{E'V}{X_{fault}} \sin\delta \right) d\delta = \int_{\delta_c}^{\delta_m} \left(\frac{E'V}{X_{postfault}} \sin\delta - P_m \right) d\delta$$

$$P_m(\delta_m - \delta_0) + E'V\left(-\frac{\cos\delta_0}{X_{fault}} + \frac{\cos\delta_m}{X_{postfault}} \right) = E'V\cos\delta_c \left(-\frac{1}{X_{fault}} + \frac{1}{X_{postfault}} \right)$$

and this allows us to solve for the critical clearing angle.

9. Calculation of Critical Clearing Time

The equal area criterion allows us to calculate the critical clearing angle but nor the critical clearing time. Since the swing equation is a non-linear second order differential equation, generally we need to us a numerical method to solve it. We can use the Euler or Modified Euler, the Runge-Kutta or any other appropriate method. The result of the application of such a technique is the variation in rotor angle with time. Figure 13 shows the response of the system described in the problem above. In this case the fault is cleared at a time of 0.64 seconds after its application. As can be seen the system is stable on the first swing. From this we can see that the initial angle is

21°, the maximum angle is about 135° and the clearing angle is about 120°. If the fault clearing is delayed to 0.68 seconds, then the result is as shown in Figure 14. Here, the system is clearly unstable.

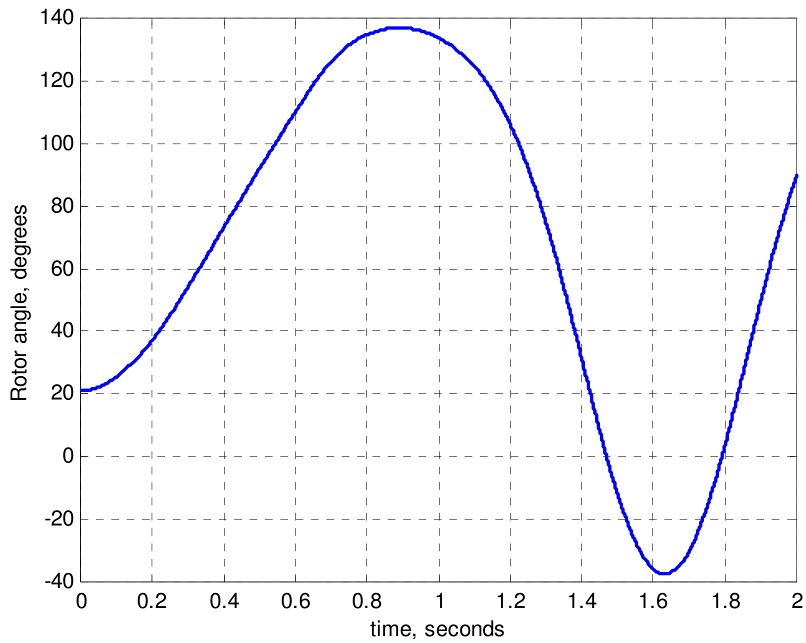

Figure 13 Stable operation

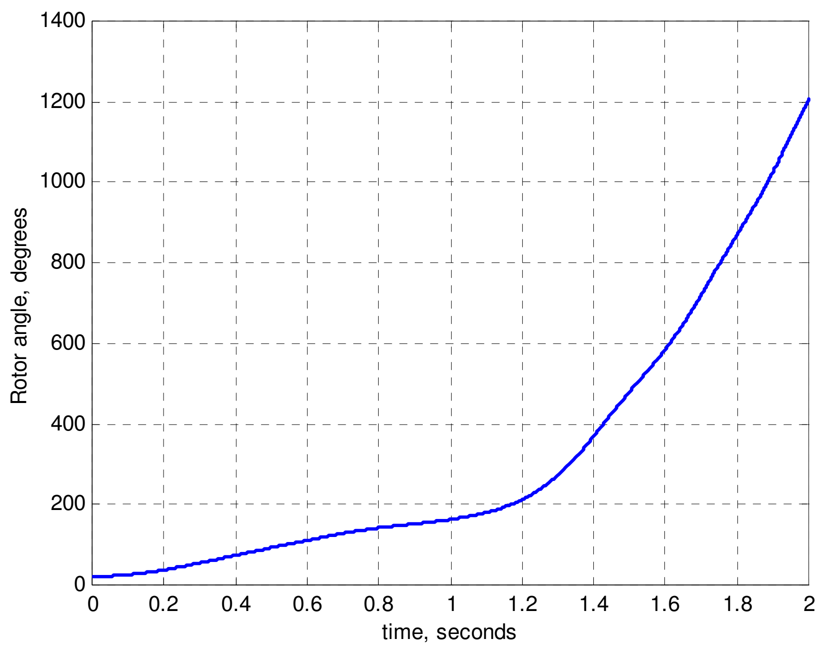

Figure 14 Unstable operation

10. Analysis of Multimachine Model

A typical power system will consist of many generators connected via a transmission network. Transient stability analysis in the case of a multimachine machine system involves investigating the response of all those machines to a given fault. Figure 15 shows a system consisting of three machines and 9 buses. A fault occurs close to bus 7 and the fault is cleared by opening that line 5 - 7.

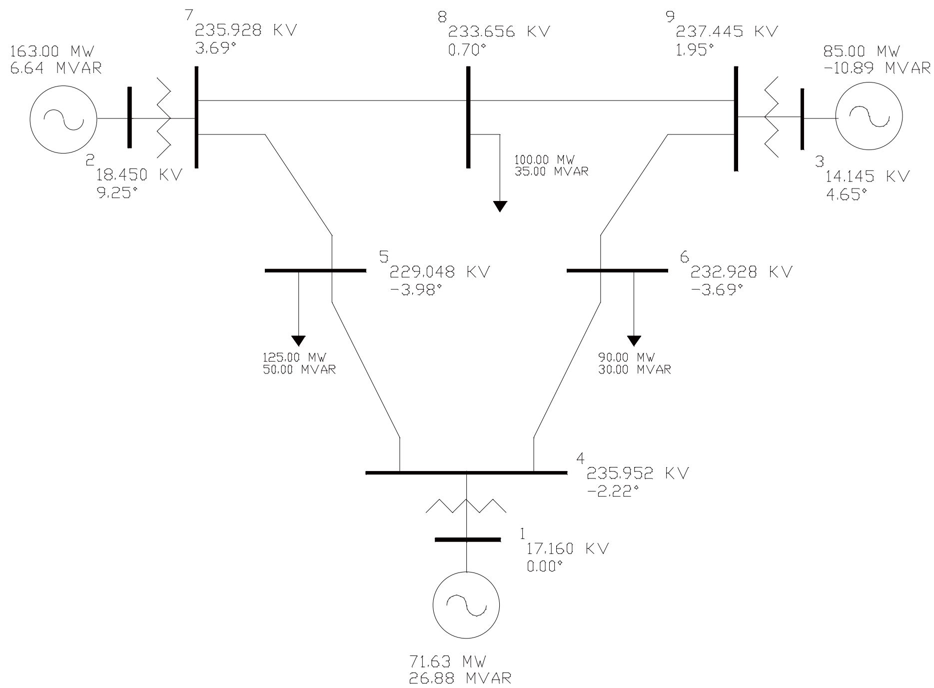

Figure 15 Three machine system

The response of the rotor angles of the three machines is shown in Figure 16. Here we can see that the rotor angles are steadily increasing. However, if we plot the rotor angle of machine 2 against machine 1 and the rotor angle of machine 3 against machine 1, then we can see that the system remains stable. This is shown in Figure 17. In this example the clearing time is 5 cycles. Figure 18 shows the generator output electrical power.

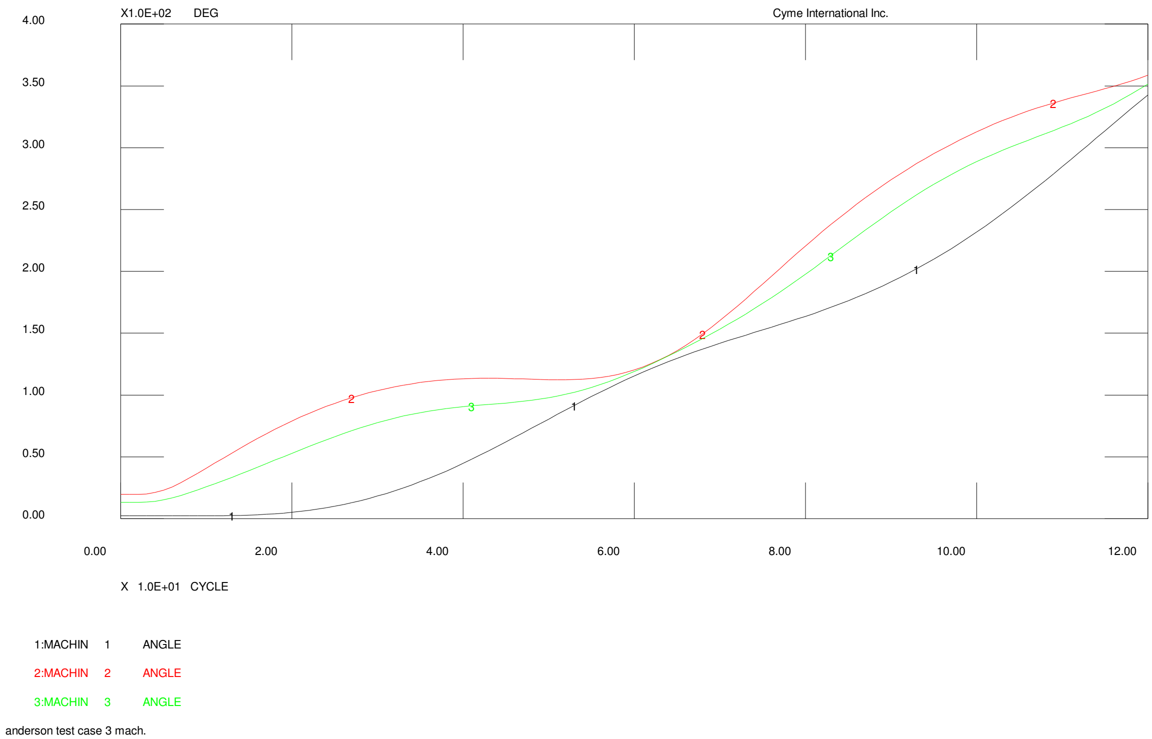

Figure 16 Absolute rotor angles

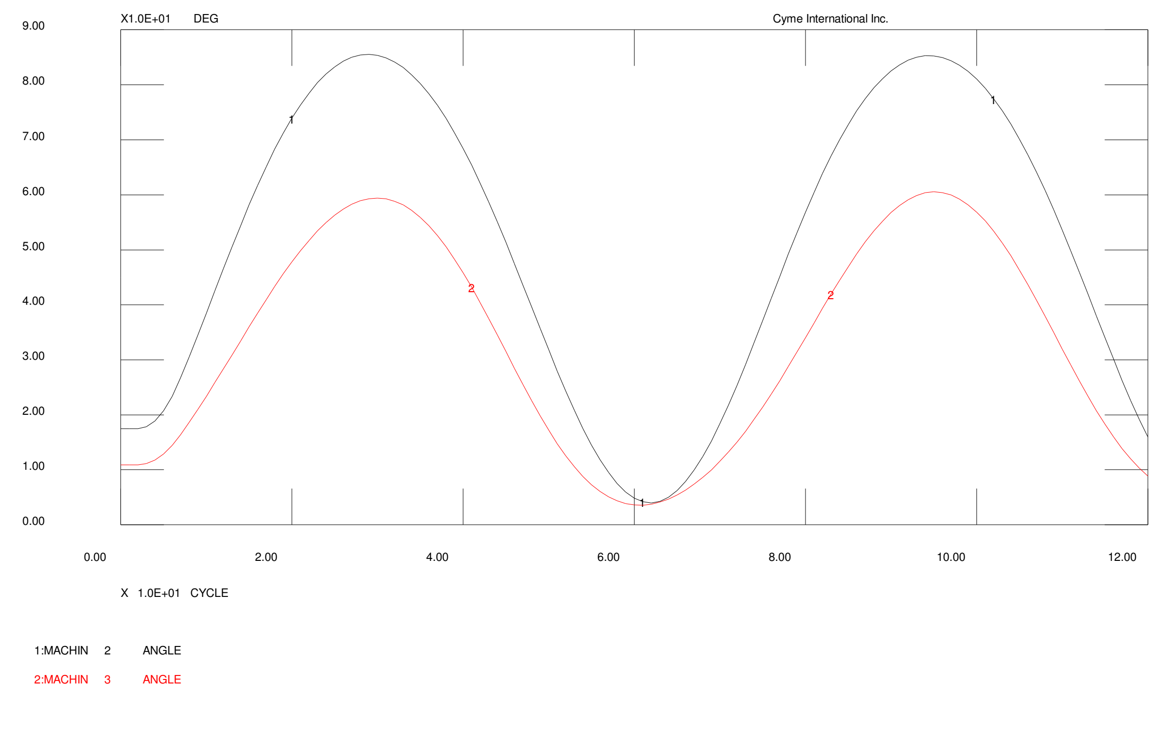

Figure 17 Relative rotor angles

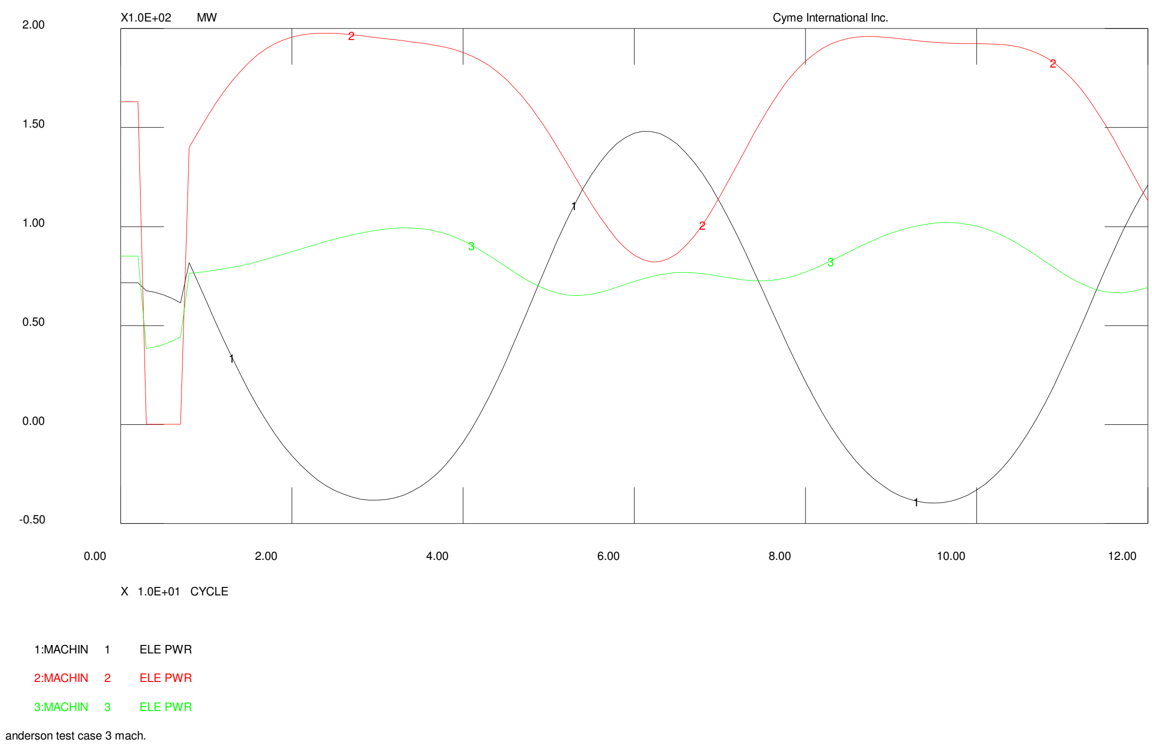

Figure 18 Generator electrical power

11. Review of Classical Model

The representation which has been used for the analysis is called the classical model. Some of the shortcoming of this model are as follows:

(a) Transient stability is decided in the first swing.
(b) Constant generator main field-winding flux linkage
(c) Neglecting the damping powers
(d) Constant mechanical power
(e) Representing loads by constant passive impedances.

Possible improvements to this representation include:

- study the transient response for more than one second as maximum swing may not be the first swing in a multibus system
- include representation of dynamic response of AVR. Also represent turbine-governor characteristics in model.
- include damping in order to improve the accuracy of the model. Also represent saturation in iron
- loads may need to be represented by constant P and constant Q, or have voltage and/or frequency dependence

12. Improvement of Transient Stability

The main factors and controls which enhance transient stability:

- Excitation systems
- Turbine valve control
- Faster fault clearing times
- Single pole operation of circuit breakers
- Minimise transformer reactances
- Series capacitor compensation of lines
- Additional transmission lines
- Load shedding
- Breaking resistors

Michael F. Conlon
March 2003